Data-Driven Anisotropic Finite Viscoelasticity with Neural ODEs

Vahidullah Tac,
Manuel K. Rausch, Francisco Sahli Costabal, Adrian Buganza Tepole

Background

Some materials exhibit a time-dependent mechanical behavior. If any material is deformed it will develop internal stresses in response to this deformation. In a material like steel, the stresses will remain constant through time, whereas some other materials, say polymers, will gradually 'relax' resulting in reduced stress over time. This is known as 'viscoelasticity' [1]. Most soft tissues in our body and most polymers, especially rubber, show viscoelastic behavior. Accurate modeling of the viscoelastic behavior of these materials is extremely important for a number of applications in the frontiers of science such as personalized surgery [2] and soft robotics [3].

Existing models of viscoelasticity use closed-form expressions to model viscoelastic behavior [1, 4]. This approach severely lacks flexibility, because a given closed-form expression is limited only to a specific range of behaviors. On the other hand, data-driven models are *formless*; they can mimic any behavior if the model is large enough [5]. Using data-driven methods can result in highly flexible models as has been illustrated for a number of applications including in modeling the hyperelastic [6, 7] and plastic [8] behavior of materials. However, blindly using data-driven methods in our models without a thorough consideration of the underlying physics is a recipe for disaster [9]. The question then becomes, ***can we use data-driven methods to model viscoelasticity in a way that physical constraints are always satisfied?*** We solve this problem by developing models of viscoelasticity using Neural ODEs; a new machine learning algorithm with some interesting properties.

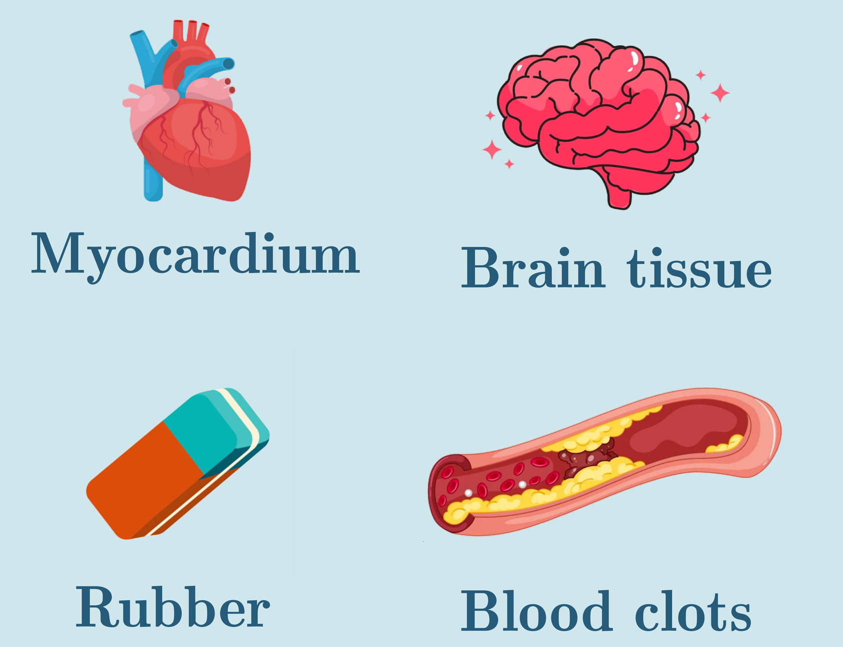

Fig. 1: Some examples of viscoelastic materials.

Physical constraints in viscoelasticity

Objectivity
Polyconvexity
2nd law of Thermodynamics
Thermodynamic consistency
Material symmetry

The mechanical behavior of viscoelastic materials is subject to a few physics-based constraints such as objectivity, material symmetries, polyconvexity, and most crucially, the second law of Thermodynamics. The second law of Thermodynamics states that for any state of stress at a point, the deformation of the material has to be such that the dissipation of energy is always positive!

The dissipation of energy in viscoelastic materials is governed by a function known as the *dissipation potential*. It can be shown that to guarantee positivity of dissipation, the dissipation potential has to satisfy some criteria, most importantly it must be a *convex function* of the stress [10].

We use Neural ODEs to enforce the 2nd law of Thermodynamics

Our approach is based on 3 key ideas:

I. Neural ODEs are monotonic.

II. If the derivative of a function is monotonic, the function itself is convex.

III. Convex functions can be used to guarantee positivity of dissipation.

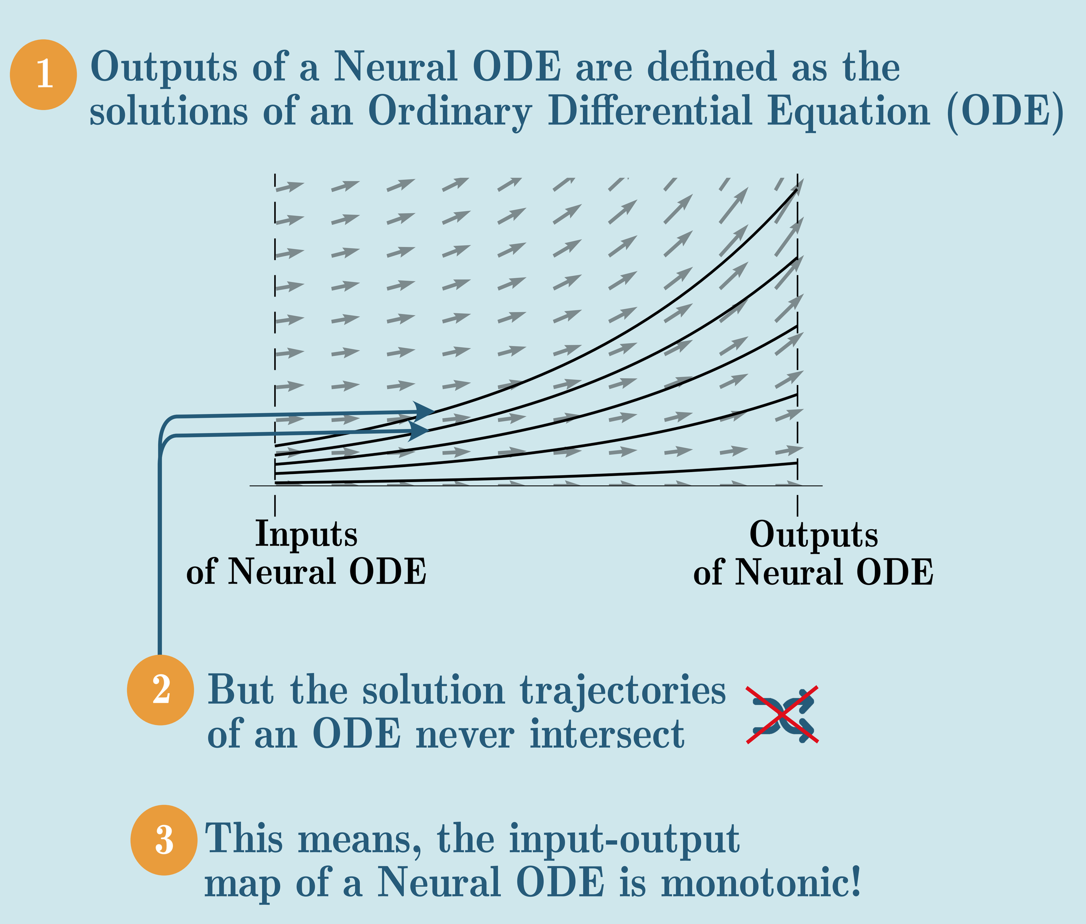

Fig. 2: How are Neural ODEs always monotonic?

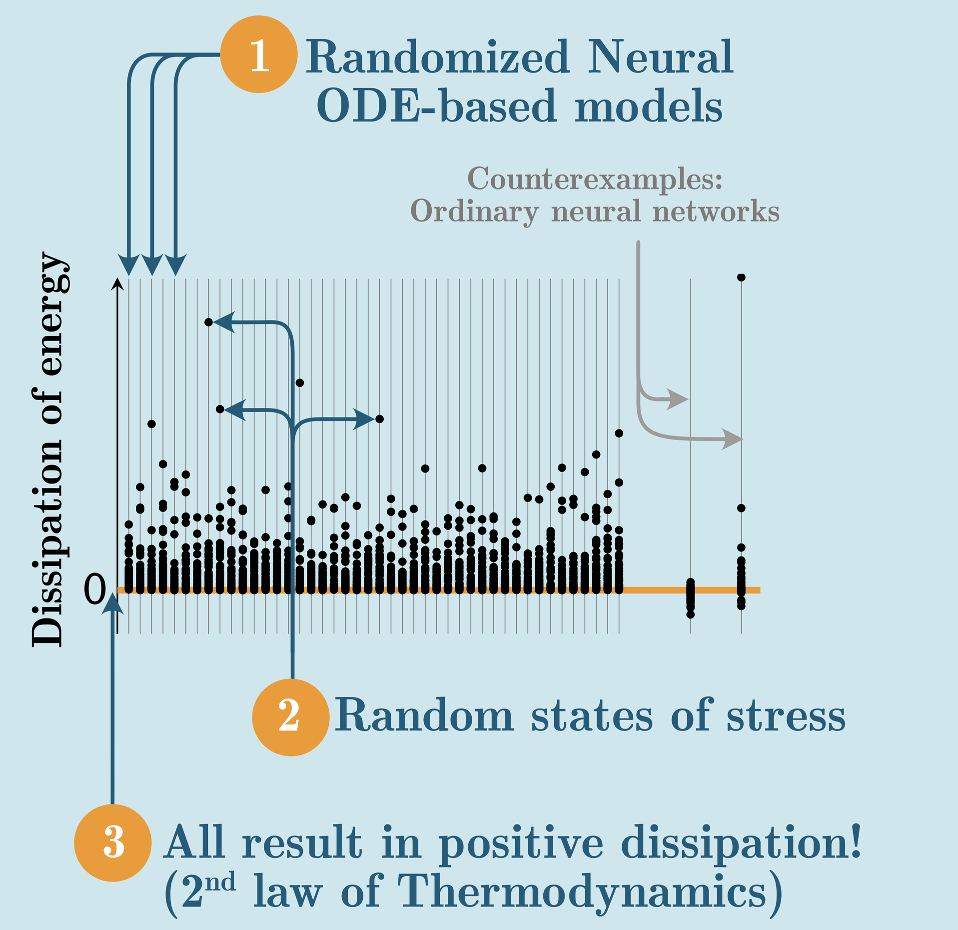

Fig. 3: The Neural ODE-based models always predict positive dissipation.

Results

We train and test the Neural ODE-based models with experimental data obtained from various synthetic and biological materials like human myocardium and rubber. The data are obtained under stress relaxation experiments.

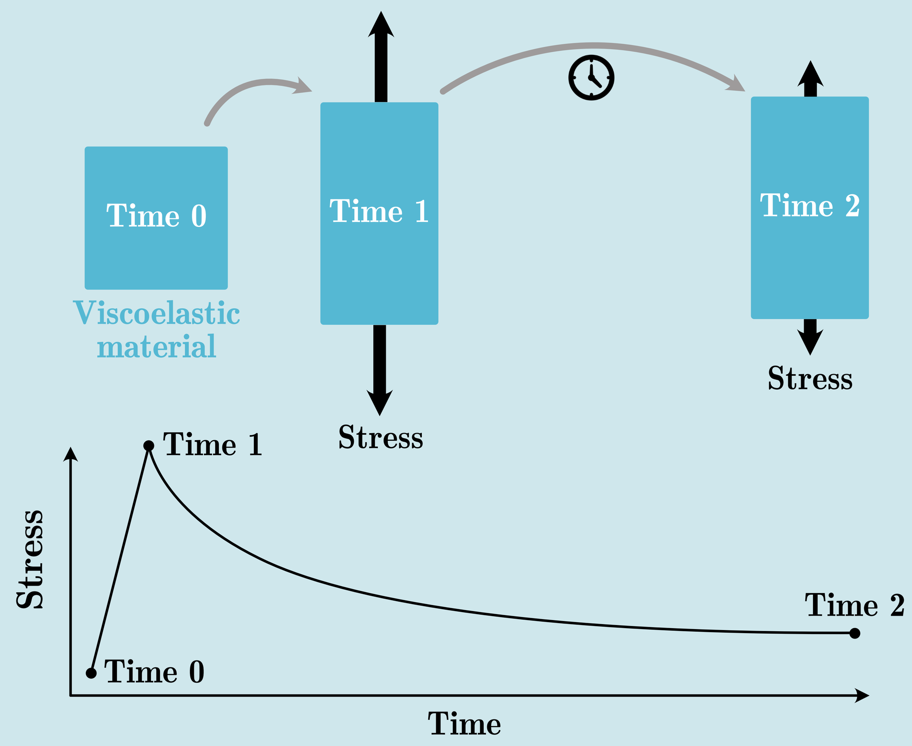

Fig. 4: Stress relaxation experiments.

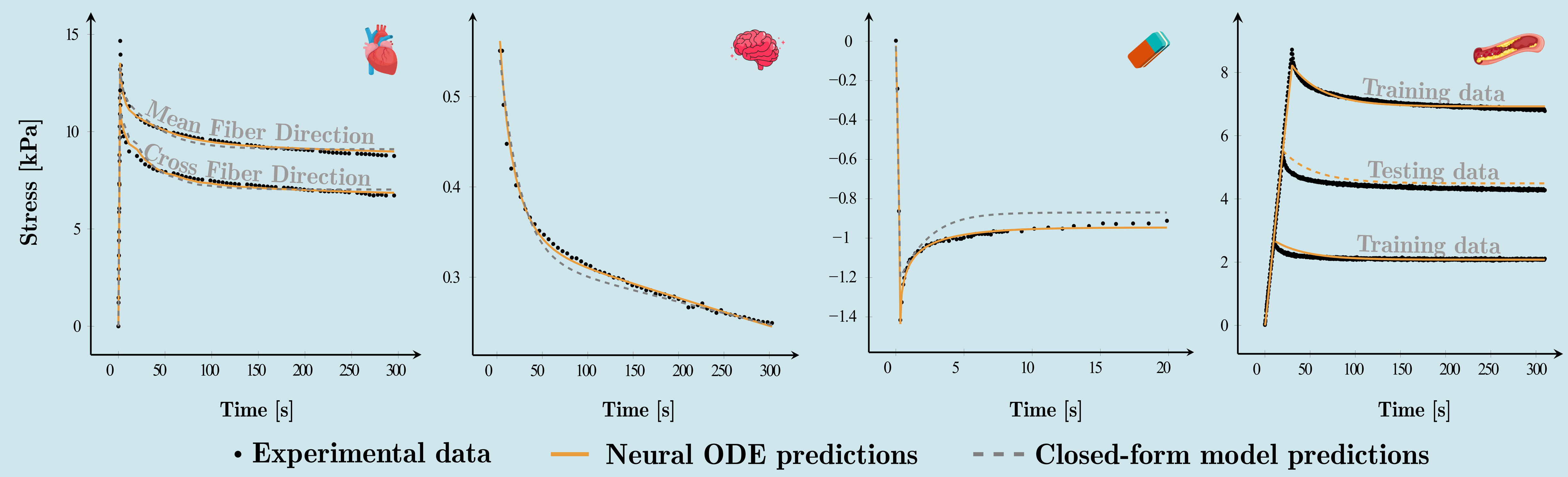

Fig. 5: Modeling stress relaxation behavior of various materials with Neural ODEs.

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